We increase our probability of survival and wellbeing by minimizing our exposure to rare, extremely negative events. In this article, we examine the computations used to predict and avoid such events and to update our models of the world and action policies after their occurrence. We also consider how these computations might go wrong in anxiety disorders and Post Traumatic Stress Disorder (PTSD). We review evidence that anxiety is linked to increased simulations of the future occurrence of high cost negative events and to elevated estimates of the probability of occurrence of such events. We also review psychological theories of PTSD in the light of newer, computational models of updating through replay and simulation. We consider whether pathological levels of re-experiencing symptomatology might reflect problems reconciling the traumatic outcome with overly optimistic priors or difficulties terminating off-line simulation focused on negative events and over-generalization to states sharing features with those antecedent to the trauma.

Addresses
1 Department of Psychology, University of California, Berkeley, CA 94720, USA
2 Helen Wills Neuroscience Institute, University of California, Berkeley, CA 94720, USA
3 Gatsby Computational Neuroscience Unit, University College London, London W1T 4JG, UK

Corresponding author: Bishop, Sonia J (abishop@berkeley.edu)

Introduction
In modern life, aversive events vary both in their frequency and severity. Shootings, terrorist incidents and plane crashes are rare, extremely negative events that might threaten our survival if experienced even just once. Avoiding exposure to such events and handling them appropriately if they occur is critical to our survival and well-being but, we argue, surprisingly hard to integrate smoothly into the course of our day-to-day lives. Here, we lay out this computational problem as a form of approximate Bayesian decision theory (BDT) [1] and consider how miscalibrated attempts to solve it might contribute to anxiety and stress disorders.

According to BDT, we should combine a probability distribution over all relevant states of the world with estimates of the benefits or costs of outcomes associated with each state. We must then calculate the course of action that delivers the largest long-run expected value. Individuals can only possibly approximately solve this problem. To do so, they bring to bear different sources of information (e.g. priors, evidence, models of the world) and apply different methods to calculate the expected long-run values of alternate courses of action. Avoiding catastrophic events poses unique difficulties above those in other situations framed in BDT because the rarity of these events renders methods that work well in more typical situations, such as model-free learning, relatively less useful. As a result, model-based processes that more efficiently re-use and extend experience, such as replay and counterfactual simulation, become especially important both before and after these events.

In part 1, we discuss the computations required to take into account the potential future occurrence of yet-to-happen rare, extremely negative events as we plan and navigate our daily lives. We consider how individual differences in these computations might confer vulnerability to anxiety. In part 2, we focus on the computations required to update our models of the world and action policies after the occurrence of rare, extremely negative events. We explore how the re-experiencing symptomatology characteristic of Post Traumatic Stress Disorder (PTSD) might be understood in the context of these computations.

Part 1: anxiety and predicting the future
The survival circuits that are the focus of this issue provide rich, hard-wired (sometimes called Pavlovian) policies that directly determine particular actions in the face of immediate mortal threat. However, waiting until threats materialize is rarely wise; maximizing our chances of survival and well-being requires estimating the probability and cost of extreme negative events and developing strategies for ameliorating or avoiding them ahead of time.

When estimating the expected value of avoidance behaviors, one should weight outcome costs by outcome
probabilities. In the case of rare, extremely negative events, the costs are so high that even small differences in probability estimates will have a huge impact on these expected values. High trait anxious children and adults produce higher estimates of the probability that future negative events (e.g. being involved in a road accident) will occur to them than do low trait anxious participants [2–4]. Such differences in probability estimates might result in increased selection of avoidance behaviors despite the associated disruption to everyday life activities.

There are a number of potential sources of these anxiety-related differences in probability estimates; these include differences in the method of estimation used, in initial biases in the estimates (priors) and in the precision of estimate calculation. The probability of rare, extremely negative events is hard to calculate precisely because similar events have rarely, if ever, been actually experienced. Thus, probability estimates are likely to be broad, with weak upper bounds. If the world is rapidly changing, that is, volatile, these bounds should be weaker still, as only very recent outcomes are pertinent [5]. In anxiety, there is evidence for difficulties in estimating environmental volatility [6,7] and increased adoption of high volatility priors [8*]. Hence, anxious individuals might have even weaker upper bounds than other individuals for probability estimates of rare, extremely negative events. One strategy for robustly avoiding catastrophic failures is to adopt a worse-case scenario ($H_\infty$ control) [9], that is, to rely on the upper bound as opposed to the mean or median of the distribution of outcome probabilities. Consistent with this, clinically anxious individuals are reported to engage in catastrophizing, focusing on worst case outcomes [10]. If anxious individuals do indeed show a combination of widened bounds for probability estimates of rare, extremely negative events and reliance on upper bounds during action selection, this might promote more frequent selection of avoidance behaviors.

If we seek to weigh up the benefits of certain behaviors (e.g. going on vacation in London) against the potential probability and cost of rare, extremely negative events (e.g. a plane crash or terrorist incident), we must calculate the long-run values of alternate actions. Long-run values take into account outcomes that might only arise several steps after the initial choice. The methods used for this are often conceived as living on a spectrum between so-called model-free and model-based computations [11,12]. Model-free and model-based methods both aim to produce appropriate policies (which specify the actions that should be taken in different situations), however, they use information from the world differently to do so. Model-free methods such as temporal difference learning [13,14] cache information during experience of the environment. They thereby create policies that are computationally straightforward to use to guide subsequent on-the-spot action selection and have the speed to be well suited to the avoidance or mitigation of extremely negative events. However, use of model-free methods to create multi-step action policies aimed at avoidance of rare, extremely negative events is heavily compromised by the reliance of these methods on past experience, given the typical absence of past experience for such events.

In contrast, model-based methods construct internal representations of alternate states of the world, and of how alternate future courses of action might play out depending on the initial state encountered [14,15]. Construction of such models of the world is informed by direct experience. However, indirect evidence such as vicarious experience or intuitive knowledge about the physical world can also be incorporated [16]. Sampling from the model can be used to play out what might transpire given selection of a particular initial action, even if that action has not been taken in real life. Thus, model-based methods can anticipate states and outcomes that have never been experienced, a characteristic of particular value for working out how to avoid rare, negative events. Such sampling can be used directly for planning [17]. However, it has also been suggested that sampling during off-line periods (such as quiet wakefulness or sleep) can be used to train model-free estimates of action values [18]. The putative benefit of this is to create model-free action policies that are fast to use but nevertheless reflect the knowledge contained within the model. However, if the model, or samples drawn from it, is biased, then not only will model-based planning be biased, but the model-free policy trained on the basis of the samples drawn will become biased too.

Biased sampling is likely to impact all of us, to some extent, but might be a particular problem for individuals at risk for anxiety disorders. Given the impossibility of exploring all potential future states, we need strategies for restricting the states we consider. It has been suggested that we focus on states that are easily available [19]. The frequency with which states have been encountered in the past is likely to impact their availability. This may result in low frequency outcomes being overlooked during the estimation of action values. However, relying on state frequency alone might be suboptimal in some situations, and it has been suggested that this might be offset by the oversampling of emotionally salient outcomes, especially those involving extreme (i.e. rare, high value) events [20**]. In line with this, emotional salience has been shown to facilitate recall of past events [21], in particular in the case of extreme events such as terrorist attacks [22,23]. That increased availability of such events might impact action valuation is indicated by findings that availability of positive and negative events during simulation and recall predicts estimates of the future probability of these events [24,25]. A current example is the reported drop in Southwest bookings the week after an engine
broke apart resulting in the death of a passenger on one of their flights.

Anxious individuals potentially oversample negative extreme outcomes and associated antecedents to a greater extent than individuals low in anxiety. In line with this, participants with high anxiety levels selectively generate more negative possible future life events than low anxious participants within a limited period of time [26]. If anxiety is linked to oversampling of negative outcomes and their antecedents, the frequency of such simulation might also moderate the extent to which estimates of the values of avoidance behaviors are influenced by sampling biases. Worry, repetitive thinking focused on future potential threat, imagined catastrophes and their possible prevention [27], is a common form of simulation of the real world. Elevated levels of worry are a defining feature of Generalized Anxiety Disorder and also characterize other anxiety disorders [28]. Anxious adults report more worry episodes and greater overall time engaged in worry [29], and anxious children report being unable to stop worrying until the focus of worry is removed [30].

Frequent, uncontrollable simulation of negative outcomes and their antecedents might contribute to the maintenance of anxiety disorders by increasing the subjective valuation, and selection, of avoidance behaviors. If anxious participants also show increased reliance on upper bounds of probability estimates for rare negative events (as discussed above), this will have a converging influence upon the overvaluation of avoidance behaviors. These behaviors, in turn, will reduce opportunities for anxious individuals to collect data showing that extreme negative events almost never occur, even if avoidance behaviors are not engaged. Therefore, there will not be the observations needed to correct estimation biases and stabilize a potentially detrimental cycle of increasing miscalibration of action value estimates and selection of avoidance behaviors. Such decision-theoretic path dependencies have been implicated in various other psychiatric contexts [31,32].

**Part 2. PTSD and replaying the past**

Despite our best efforts, extremely negative events do occasionally occur. If such an event is survived, the balance of planning activities should shift toward avoiding the event being experienced again. This is both because the events occurrence might contain information useful for avoiding similar events in the future and because there might be autocorrelation in the occurrence of extremely negative events (e.g. when new predators enter an environment [33]).

Off-line replay of prior experiences and simulation of counterfactual actions and associated outcomes provide a means to update action values following the occurrence of an extremely negative event [18]. It has been argued that previous states should be prioritized for replay based on how much that replay would change value estimates [34**]. One way to accomplish this is by tagging states based on how much their successors value has changed [35]. If change in value estimates determines priority for replay, the astronomically large discrepancy in outcome value occasioned by the occurrence of a rare, extremely negative event would be expected to result in prioritized replay of that events antecedents (see Figure 1).

By replaying the states that preceded a rare, extremely negative event, we can ascribe more negative values to these states and the actions selected within them that led up to the events occurrence. Equally, actions that were not taken can be simulated, together with the possible outcomes of these actions. Should similar states be encountered again, the model-free system can use the updated action values to choose swiftly a safer course of action. However, this may not be entirely straightforward. Specifically, exposure to a rare, extreme event might increase the salience and availability of similar outcomes and increase the probability, from an otherwise negligible level, of such outcomes being simulated following various actions. This might result in many courses of action being evaluated more negatively than before the experience of the extreme negative event.

Findings from the traumatic stress literature indicate that most individuals do indeed replay the antecedents of extreme negative events after their occurrence. Following extreme negative (also termed ‘traumatic’) events, such as motor-vehicle accidents, over 50% of individuals report intrusive recollections, flashbacks, or nightmares up to three months following the event [36,37]. These phenomena are collectively referred to as re-experiencing symptomatology. It is also common for people to ruminate repeatedly on their experience, thinking about the events causes and ways that it might have been prevented, for example, ‘I was running late so I cut through town, if I had gone the long way round . . . ’ [36,38,39].

Some degree of re-experiencing, rumination and counterfactual thinking about the past might be functional in the wake of an extremely negative event, or trauma. Indeed, psychological accounts have argued that repetitive thinking in the wake of a traumatic event might be important for resolving the discrepancy between the event and pre-existing core beliefs or assumptions. Horowitz [40] describes this process as ‘cognitive processing’ and Janoff-Bulman [41] describes it as ‘integration’, resulting in what Tedeschi and Calhoun [42] describe as ‘post-traumatic growth’. Critically, whereas re-experiencing symptoms decay rapidly over time for some individuals, for others they remain frequent and cause significant levels of distress and disruption to everyday life. Researchers have struggled to identify aspects of the re-experiencing process or content that predict
A replay and simulation account of PTSD. When a traumatic event is experienced, the difference in value between the expected and actual outcome is calculated (box 2). Given the high negative value of the traumatic event and its low prior probability of occurrence, a large negative prediction error (i) will be experienced. This prediction error triggers the replay of the antecedent state and action (box 3). During this replay, the antecedent state and action will be ascribed a more negative value. Alternate actions will also be simulated in order to identify a better counterfactual course of action from the antecedent state. The action values employed when the initial decision was made (box 1) will be updated. Pre-trauma, the sampling, and hence consideration during action valuation, of traumatic outcomes is unlikely. Post-trauma, the increased salience and availability of such outcomes will make their sampling more probable. Even very small increases in the estimated probability of actions leading to the same, or another, traumatic outcomes will result in substantial negative revision of action values given the large negative value of such outcomes. Replay and simulation will continue until a counterfactual action is found with a sufficiently large expected value (Q(Sa, a)) or until the prediction errors (i) resulting from the replay and simulations have sufficiently diminished. Increased estimates of the probability of traumatic outcomes and associated downward revision of action values may result in many actions being considered in search for one with an acceptable value. Stopping criteria (thresholds for Q(S, a) or i) may vary across individuals and may potentially be influenced by external factors (e.g. novel stressors). Other states that share perceptual or semantic features with the state antecedent to the traumatic event will likely also be replayed or simulated and their associated actions may be ascribed a more negative value to the extent that shared features increase the availability, during simulation, of traumatic outcomes (box 4). The value of the state immediately antecedent to the traumatic event (S1) will entail that the values of states and actions (starting from S1, A1, i) before that state will also need to be updated [34**,35]. This will result in the replay and re-evaluation of increasingly earlier states and actions (box 5). Generalization of values to similar states and the simulation and re-evaluation of alternate courses of action will also occur from these more distant antecedents. This cycle of replay will continue, with gradual discounting as more temporally distant states are revisited, until prediction errors (i) sufficiently diminish or until counterfactual actions can be found at each temporal point in the antecedent chain and at each level of generalization with sufficiently high expected values. This process of replay of chosen action paths and simulation of alternate action paths might be phenomologically experienced as intrusive thoughts, dreams, rumination and counterfactual reasoning. Red text is used to signify points where individual differences might confound vulnerability to elevated PTSD symptomatology. (a) Individuals may differ in stopping criteria; for example, individuals vulnerable to PTSD might be reluctant to take actions with even the slightest possibility of future catastrophe or might have a lower tolerance for negative changes in action or state values. (b) A reduced rate of discounting of the prediction error as more temporally distant antecedents are considered and (c) a shallower generalization slope when re-valuing states or actions that resemble those antecedent to the trauma are likely to increase the number of states and actions replayed resulting in higher levels of re-experiencing, avoidance and hyper-vigilance.

post-traumatic growth versus disorder [43]. Here, we suggest that the re-experiencing symptomatology and negative cognitions (e.g. rumination and counterfactual reasoning) observed following an extreme negative event can be computationally operationalized in terms of replay and simulation. In the rest of this section, we consider how this operationalization might shed light on the potential determinants of healthy versus dis-ordered responses to the experience of an extremely negative event.

Several psychological accounts of PTSD posit that individuals with rigid beliefs about the positive nature of the world are more likely to experience PTSD after a traumatic event due to their assumptions or schema about the world being unable to flexibly accommodate or integrate the traumatic experience [41,44]. According to BDT, if an individual has a much lower prior expectation of the occurrence of extremely negative events, this will generate a larger discrepancy between the value of the expected outcome and the value of the actual outcome (the traumatic event), strongly prioritizing the replay of the events antecedent states and actions [34**,35] and likely resulting in greater re-experiencing symptomatology. If rigidity is associated with an unwillingness to update state and action values to make them more negative, then negative prediction errors will stubbornly persist.
Individuals who go on to develop PTSD endorse more negative world views in the immediate aftermath of trauma [45]. Moreover, elevated pre-trauma levels of anxiety and depression have been found prospectively to predict levels of post-traumatic stress symptomatology [46]. Since anxiety and depression are linked to negative, not positive, biases in beliefs, interpretations of ambiguous events and judgements about the future [47–49], it seems unlikely that the magnitude of the prediction error occasioned by experiencing a traumatic event is a key predisposing factor, at least for these individuals. Indeed, there is little empirical evidence that pre-trauma possession of optimistic priors confers vulnerability to PTSD.

There are other ways in which individual differences might influence extent of engagement in replay and simulation. One possibility is that individuals with a pre-trauma history of anxiety or depression might be more prone to difficulties with terminating disadvantageous replay or simulation processes. Such stopping difficulties might confer vulnerability to PTSD as well as to anxiety disorders (as described in Section 1) and depressive disorders. In line with this, both anxiety and depression are characterized by elevated levels of repetitive thoughts (worry and rumination) [50]. Further, pre-trauma engagement in repetitive thinking is a significant predictor of post-trauma levels of PTSD symptomatology [51]. In the anxiety literature, difficulty finding a potential course of action with a sufficiently positive expected value has been associated with increased uncontrollability of worry [52]. Post-trauma, inability to identify one or more counterfactual courses of action with high enough expected values to terminate simulation and replay might similarly be linked to elevated PTSD symptomatology (Figure 1).

An increased propensity for repetitive thinking might be further compounded by a disposition to over-generalization [53]. Clinical accounts of PTSD describe how everyday sounds, like a balloon popping or a car backfiring, that resemble a gunshot can lead to extreme physiological and emotional responses in individuals for whom the traumatic event involved combat or gun violence. This has led to suggestions that over-generalization might be a key vulnerability factor in PTSD [38]. In terms of the replay and simulation framework put forward here (see Figure 1), when an individual uses replay and simulation to update the value of states and actions that preceded a traumatic event, a key issue will be determining how far to go in also updating the value of other states and actions that share features with the antecedent states and actions. Other states may be related to antecedent states at a very concrete level (e.g. a similar looking street-corner to where the accident occurred) or at a very abstract level (e.g. any form of transportation in which you are not in control). Both selection of an abstract level and, at any given level, adoption of less steep (dis)similarity gradients might result in larger numbers of states and actions being re-evaluated both off-line and on-line when the individual encounters a state that shares features with a state antecedent to the traumatic event. Future planning would suffer from a similar problem.

There is strong evidence linking both forms of overgeneralization described above to anxiety, depression and PTSD. Patients with GAD, Panic Disorder, and PTSD have been shown to generalize from a conditioned stimulus across a wider range of perceptually similar shapes than healthy controls [54,55,56]. In addition, over-general autobiographical recall has been shown to characterize both patients with depressive disorders and individuals with a history of trauma [57]. Further, levels of rumination have been shown to increase the influence of over-general memory on both future depressive and post-traumatic stress symptomatology [58,59]. Trauma-analog studies have also reported that participants asked to ruminate abstractly, versus concretely, after viewing a traumatic video show both prolonged negative mood and more negative intrusions [60,61].

In addition to re-experiencing, PTSD is also characterized by hyper-arousal and avoidance behaviors, together with other symptoms [28]. Within a replay and simulation account of PTSD, both over-generalization and going further back in the chain of antecedent states as part of the replay process (see Figure 1) will result in more states and associated actions being re-evaluated as potentially dangerous. This, in turn, could lead to an increased sense of current threat, associated physiological reactivity and avoidance of multiple situations.

One important question for future research is whether individuals with a prior history of anxiety or depression are equally likely to show over-generalization between perceptually similar stimuli or states and over-abstract levels of simulation, or if the former might be more associated with anxiety and the latter with depression. In addition, the extent to which individuals vary in stopping criteria for terminating replay and simulation, and the role of this in anxiety, depression and PTSD, remains to be established.

Conclusion
In order to survive and maximize our wellbeing, we need to weigh up actions aimed at avoiding life threatening events against the pursuit of rewarding activities and avoidance of more minor aversive outcomes or losses. Here, we have outlined the computational processes involved in action valuation both in advance of, and subsequent to, the occurrence of rare, extremely negative events, and discussed how both anxiety and PTSD might be understood within this computational framework. Future work would valuably extend our computational analysis to consideration of potential neurobiological markers of disease risk. Given the putative role of hippocampal function in off-line
model-based simulation processes [62–64] and evidence for hippocampal dysfunction in both anxiety [65–68] and PTSD [69–71], this is a particular structure of interest.

Conflict of interest statement
Nothing declared.

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References and recommended reading
Papers of particular interest, published within the period of review, have been highlighted as:

- of special interest
- of outstanding interest


This paper considers how we sample from memory when making decisions involving extreme (i.e., rare, high value) negative or positive outcomes. According to the account put forward, sampling low frequency outcomes that are readily available as a result of their emotional salience is often optimal given cognitive constraints. The effect of sampling bias on estimation error is held to be offset by a reduction in sampling variability.

34. Mattar MG, Daw ND: Prioritized memory access explains planning and hippocampal replay. bioRxiv 2018. Preprints a theoretical account for how the states replayed during policy updating are prioritized. The theory proposes that priority for replay
should be given to states that have the largest potential to change expected utility if included in the policy update. Under this theory, prioritized states are often those antecedent to outcomes that generate the largest prediction errors. The theory accommodates findings from the hippocampal replay literature, such as the change in reverse-replay frequency following increases or decreases in reward.


